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PREDICTING SUCCESS IN GEOMETRY, ALGEBRA II, AND SENIOR MATHEMATICS AT LARNED HIGH SCHOOL

by 655

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INTRODUCTION

Background Information: Larned, Kansas, in 1963, was a city of approximately 5,500 people located in south central Kansas. It served a rural area of 9,500 people in Pawnee County. Larned was originally a military fort located on the Santa Fe Trail. With a decline of the frontier, Larned changed to an agricultural community. In 1914 the Larned State Hospital was established for the treatment of the mentally ill. This hospital serves the western half of Kansas.

The Larned School District covered an area larger than the city limits. The assessed valuation was twelve million dollars and the annual budget was four hundred fifty thousand dollars. The school system operated on a 6-3-3 plan and had an enrollment of 1,248 students. Larned School System had a faculty of sixty members in 1963.

National Trend: Across our nation there had been a growing recognition of the need of accelerated mathematics programs in high schools. One of the basic ideals of our educational systems is that we should provide for individual needs and differences in our school population. It is only logical to assume that one of the ways to meet these needs and differences is through an accelerated program of study. These needs were beginning to be recognized in the Larned System.

<u>Situation At Larned High School</u>: The original purpose of this study was to develop criteria for setting up an accelerated mathematics program for the Larned Senior High School. At the end of the school year of 1962-1963, the Superintendent of Schools at Larned, Kansas, suggested that the mathematics department be revamped for the purpose of introducing an accelerated mathematics program. After several months of work on this program, it was set aside during the summer months. During this time there was a change in administration. At the start of the 1963-1964 school year, work on the mathematics program was continued. At this time, the new administration was briefed on the suggested program. The new Superintendent let it be known that he was against accelerated programs of any type. He felt that students should not be grouped in any manner but should be given a free opportunity to choose any class they desire regardless of their ability. His reason for this was that because of the size of the school (344), he did not feel the needs of both college bound students majoring in areas requiring a great deal of mathematics and college bound students entering areas not requiring a great amount of mathematic skills could be met. He felt that this "segregation" of students would damage their personalities.

It was felt that there was still a need for improving the mathematics program. Discussing this with various fac-

ulty members and through readings on the subject, it was found that by using statistical methods the mathematics program could be improved. The methods used were those of predicting grades for Larned High School mathematics classes.

With the recent emphasis on mathematics and engineering fields in our nation, there had been a tendency for students to be "directed" into these fields. Larned students were no exception to this idea. Parents of many students "directed" their sons or daughters into advanced mathematics classes without regard for their abilities. As a result of this "direction", many students met failure that could have easily been avoided by proper guidance.

Statement of the Problem: As the mathematics program was set up, students often entered classes without any idea of what to expect as far as abilities needed and in some cases actual course content. There was a felt need on the part of the students and faculty for predicting of grades in mathematics. This was shown by the number of students that either failed or withdrew with a failing grade in the various mathematics classes. For example, of the total number of students enrolled in geometry for the school years 1961-1962, 1962-1963, and 1963-1964, 20.9 per cent failed or withdrew with failing marks. In algebra it was 18.1 per cent and in senior mathematics the total was 12.8 per cent of the total enrolled.

Purpose of the Study: The main purpose of this study was to limit the misplacement of students in the mathematics program of Larned High School by using statistical methods of predicting grades in geometry, algebra II, and senior mathematics.

Suggested Uses of Results: This information would give the student knowledge of his probable success in mathematics classes in Larned High School. This should reduce the number of failures in the mathematics classes. The study could also be used by the school to provide additional needed information for changes in the mathematics program. There was a feeling that some changes needed to be made to meet the needs of students in the area of mathematics. The results might show that grouping of pupils in mathematics classes at Larned High School was desirable.

Definitions of Terms Used: The following list of terms were defined for the clarification of the problem as they applied to Larned High School.

1. Geometry - A sophomore mathematics course "that treats of the properties and relations of plane figures (such as angles, triangles, polygons, circles) which can be drawn with ruler and compasses."[1]

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[1]Glenn James and Robert C. James, Mathematics Dictionary, p. 175.

2. <u>Algebra I</u> - A freshman mathematics course which was,

> that portion of mathematics whose primary consideration is the representation of numbers by symbols and the study of the rules of their operation inside a given set by means of the few elementary operations (addition, subtraction, multiplication, division).[1]

3. <u>Algebra II</u> - A junior mathematics course which was an extension of algebra I.

4. <u>Senior Mathematics</u> - A senior mathematics course which consisted of trigonometry, analytic geometry, and college algebra.

5. <u>Accelerated Mathematics Program</u> - A program that divides the mathematic students into two or more classes to meet the abilities of the students. The purpose is to allow the students with special ability to work to their full capacity and not be stymied by the ability of other students unlike their own.

6. <u>Correlation Coefficient</u> (r) - "A measure of the degree of relationship, or 'going-togetherness', between two sets of measures for the same group of individuals."[2] It ranges from .00 denoting complete absence of relationship, to 1.00, denoting correspondence, and may be either positive or negative.

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[1] <u>The Encyclopedia Americana</u>. Vol. I. p. 381D.

[2] Roger T. Lennon, <u>Test Service Notebook</u>, p. 2.

7. Regression Equation - The formula for making predictions of scores from one or more variables.

8. Standard Deviation (s_x) - "A measure of the variability or despersion of a set of scores. The more the scores cluster around the mean, the smaller the standard deviation."[1]

9. Standard Error of Estimate (s_{xy}) -

> "An estimate of the magnitude of the 'error of measurement' in a score, that is, the amount by which an obtained score differs from a hypothetical true score. The standard error is an amount such that in about two-thirds of the cases the obtained score would not differ by more than one standard error from the true score. The larger the standard error of a score, the less reliable the measure."[2]

10. Variable - "A quantity which can take on any of the numbers of some set."[3]

Limitations of the Study: In making any interpetations of this study, the reader should consider the following basic limitations:

1. The only students used were those of Larned Senior High School, Larned, Kansas, for the academic years 1961-1962, 1962-1963, and 1963-1964.
2. The test information used was from tests administered in the Larned School System.

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[1] Roger T. Lennon, Test Service Notebook, p. 5.

[2] Ibid.

[3] Glenn James and Robert C. James, Mathematics Dictionary, p. 412.

3. The predictive criteria available were limited in number.

4. The population of the variables was limited in its size.

REVIEW OF LITERATURE

It was not until after the beginning of the twentieth century that much statistical work was done in the field of prognostic testing. Before this time the problem of predicting achievement was based on the principle that individuals would perform in the future as they had in the past.

Students were often enrolled in courses in which they had no chance for success because of lack of certain abilities. Often some capable students would avoid courses that they felt would be difficult, even if these courses were needed for preparation for college. These types of situations could be somewhat reduced by proper guidance based on the statistical prediction of future success.

Many studies have attempted to determine what single criterion or what combination of criteria were the best predictors of future success in mathematics courses. These studies were published in educational and mathematical periodicals and in test manuals of certain tests. Also, there were unpublished studies made by individuals seeking a graduate degree.

One of the first attempts to measure mathematical ability was made in 1916 by Agnes Rogers.[1] It consisted of

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[1] Agnes L. Rogers, "Experimental Tests of Mathematical Ability and Their Prognostic Value," Teachers College Contribution to Education, No. 190., 1918.

a battery of tests which included arithmetic, algebra, and geometry.

Joseph Orleans[1] was one of the first to deal with prognosis of future success in high school mathematics. In the four groups in algebra he studied, he found correlations varied from .80 to .89 between achievement tests and the teacher's marks. For the fourteen groups studied in geometry, he found the correlations varied from .73 to .96 between achievement tests and the teacher's marks. The achievement test used for algebra was the _Columbia Research Bureau Algebra Test I_; for geometry it was the _Orleans Plane Geometry Achievement Test I_.

Cooke and Pearson[2] in a study of 195 pupils investigated the prognostic value of the _Orleans Geometry Prognosis Test_, _Terman Group Test of Mental Ability_, and teacher's marks in algebra for geometry. Their conclusion was the most accurate single predictor for geometry was teacher's marks in algebra (r=.546).

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[1]Joseph B. Orleans, "A Study of Prognosis of Probable Success in Algebra and in Geometry," _The Mathematics Teacher_, 27:225-246 May, 1934.

[2]Dennis H. Cooke and John M. Pearson, "Predicting Achievement in Plane Geometry," _School Science and Mathematics_, 24:872-878 November, 1933.

Lee and Hughes[1] found the best predictor of marks in algebra or geometry was the teachers rating of previous mathematical ability. The correlations of teachers rating of mathematical ability with algebra was .59; with geometry it was .42. This supported a study by Ayres[2] in which he found that the correlation between teachers estimate of mathematical ability and algebra was .634.

In two separate studies by Douglass[3] and by Dickter[4] three criteria were considered for predictive value of algebra. These criteria included the intelligence quotient, prognostic tests, and teachers' marks. Of these they found that prognostic tests were the better predictor of success in algebra. Of the three, the intelligence quotient had the least predictive value. As would be expected, a combination of all three criteria gave a more accurate prediction than any single criterion.

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[1] J. Murray Lee and W. Hardin Hughes, "Predicting Success in Algebra and Geometry," School Review, 42:188-196, March, 1934.

[2] G. H. Ayres, "Predicting Success in Algebra," School and Society, 39:17-18, January 6, 1934.

[3] Harl R. Douglass, "The Prediction of Pupil Success in High School Mathematics," The Mathematics Teacher, 28:489-504, December, 1935.

[4] Richard Dickter, "Predicting Algebraic Ability," School Review, 41:604-606, October, 1933.

In a study conducted by Layton[1] it was found that eighth grade mathematics grades were better predictors than intelligence quotients, eighth grade achievement test results, or prognostic test results. He found a correlation of .82 between eighth grade mathematics and ninth grade algebra.

Rezac[2] concluded after comparing the Stanford Achievement Test, Differential Aptitude Test, grades received in eighth grade mathematics, and the Henmon-Nelson I.Q. test, that the most accurate predictor for ninth grade mathematics grades was grades achieved in eighth grade mathematics. The poorest predictor was I.Q. scores.

Jacobs[3], Assistant of Appraisal Services, Cincinnati Public Schools, conducted a study to evaluate the effectiveness of certain aptitude and achievement tests in predicting academic success in the Cincinnati public high schools. A sample of 595 pupils was used for the study.

The correlations between the total grade point averages

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[1] R. B. Layton, "A Study of Prognosis in High School Algebra," Journal of Educational Research, 34:601-605, April, 1941.

[2] James L. Rezac, Predicting Success of Ninth Grade Mathematics Students in Manhattan Junior High School. Unpublished Master's Report, Kansas State University, Manhattan, Kansas, 1962. p. 34.

[3] James Jacobs, "Aptitude and Achievement Measures in Predicting High School Academic Success," The Personnel and Guidance Journal, 37:334-341, January, 1959.

(G.P.A.) for seniors and the Numerical Ability of the Differential Aptitude Tests (D.A.T.) was .613 for boys and .686 for girls; total G.P.A. with Mechanical Reasoning of the D.A.T. was .310 for boys and .437 for girls; total G.P.A. with the Arithmetic Proficiency of the Metropolitan Achievement Tests was .657 for boys and .716 for girls.[1]

Jacobs had the following to say regarding the higher relationship among girls,

> a probable explanation lies in the fact that boys represent a more homogeneous group than do girls. Since correlation is a function of group variability, the correlations would be expected to be somewhat higher among girls than among boys. A possible explanation of this is the larger number of drop-outs largely are among the boys of lower academic caliber, the remaining group is not only relatively homogeneous but also represents the academically better pupils on the whole.[2]

Correlations were also found for mathematics grades with the Numerical Ability of the D.A.T., Mechanical Reasoning of the D.A.T., Terman-McNemar Test of Mental Ability, and the Arithmetic Proficiency test of the Metropolitan Achievement Tests. For boys the correlation were .495, .271, .433,

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[1]James Jacobs. "Aptitude and Achievement Measures in Predicting High School Academic Success," The Personnel and Guidance Journal, 37:334-341, January, 1959.

[2]Ibid. p. 335.

and .610, respectively. Respectively for girls, they were .654, .506, .622, and .671.[1]

The data from the Differential Aptitude Tests Manual shows that, "the Numerical Ability test clearly shows the greatest predictive power for mathematics grades."[2] The correlations between mathematics and Numerical Ability of the D.A.T. for boys ranged from .27 to .65, with a median correlation of .47. The correlations for girls ranged from .25 to .71, with a median of .52.[3]

Writing in the Mental Measurements Yearbook, Benno G. Fricke, Assistant Professor of Psychology, University of Michigan, had the following to say in this review of the National Merit Scholarship Qualifing Test,

> Whether the SRA-NMSQT total score will predict school and college grades as well as the total score from other academic ability tests remains to be seen. The reviewer would be surprised to find that its predictive validity is significantly higher or lower than other tests.[4]

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[1]James Jacobs. "Aptitude and Achievement Measures in Predicting High School Academic Success," The Personnel and Guidance Journal, 37:334-341, January, 1959.

[2]George K. Bennett, Harold G. Seashore, and Alexander G. Wesman. Differential Aptitude Tests Manual. 1959, p. 37.

[3]Ibid. pp. 42-43.

[4]Oscar K. Buros (Editor), The Fifth Mental Measurements Yearbook, 1959. p. 43.

The *Encyclopedia of Educational Research*[1] sums up the studies of prognosis of success in mathematics by saying that achievement in mathematics may be predicted with only a fair degree of accuracy and that it can be predicted best by a combination of several criteria. The order of accuracy of prediction of mathematical achievement at the secondary level seems to be:

> (a) good prognostic tests, (b) mathematics marks for the previous year, (c) intelligence quotient, (d) mental age, (e) achievement tests in arithmetic and algebra, and (f) average mark in previous years.[2]

The preceding review of literature should help put the present study in its proper prespective. The results of the previous studies might be compared with the present study.

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[1]Walter S. Monroe (Editor), *Encyclopedia of Educational Research*, 1950. p. 722.

[2]*Ibid.*

RESEARCH METHODS USED

The writer of this study spent the 1961-1962, 1962-1963, and 1963-1964 school years teaching at Larned Senior High School in Larned, Kansas. By the end of the school year of 1961-1962, the writer noticed the apparent misplacement of some students in the mathematics program. At that time he decided to obtain additional data for the purpose of determining a method of selection and prediction which would reduce the percentage of pupil misplacement. This method of selection must, in essence, be a way of predicting future success in geometry, algebra II, and senior mathematics.

In attempting to do this the writer considered the following methods of prediction with ungrouped data:

1. Simple correlation coefficients.
2. Multiple correlation coefficients.
3. Linear regression equation.
4. Multiple regression equation.
5. Standard Deviation.
6. Standard error of estimate.

The data for this study came from the cumulative records of each student and from the records kept by the mathematics department.

The most common method of showing a relationship between two variables is the coefficient of correlation. The

coefficients of correlation in this study were computed by using the following formula:[1]

$$r_{xy} = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{[N\Sigma X^2 - (\Sigma X)^2][N\Sigma Y^2 - (\Sigma Y)^2]}}$$

where N is the size of population of the variables and X and Y are the considered variables.

The equation for deriving the multiple correlation coefficient for three variables used in this problem was the following:[2]

$$R^2_{1.23} = \frac{r^2_{12} + r^2_{13} - 2\, r_{12}\, r_{13}\, r_{23}}{1 - r^2_{23}}$$

where r is simple correlation coefficient.

The following method was used for deriving the linear regression equation:[3]

$$\Sigma X = aN + b\Sigma Y$$

$$\Sigma XY = a\Sigma Y + b\Sigma Y^2$$

where X and Y are the variables and a and b are the derived coefficients of the variables which will be substituted into the solved linear equation:

$$X^1 = a + bY$$

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[1]Henry E. Garrett, Statistics in Psychology and Education, p. 143.

[2]Joy P. Guilford, Fundamental Statistics in Psychology and Education, p. 401.

[3]Murray R. Spiegel, Theory and Problems of Statistics, p. 242.

where X' is the predicted score.

The following methods were used for computing the multiple regression equation for three variables:[1]

$$\Sigma X = aN + b\Sigma Y + c\Sigma Z$$

$$\Sigma XY = a\Sigma Y + b\Sigma Y^2 + c\Sigma ZY$$

$$\Sigma XZ = a\Sigma Z + b\Sigma YZ + c\Sigma Z^2$$

where X, Y, and Z are the variables and a, b, and c are the derived coefficients used in the solved equation:

$$X' = a + bY + cZ$$

where X' is the predicted score.

The standard deviations for the considered variables were calculated by the following formula:[2]

$$S_x = \sqrt{\frac{\Sigma X^2}{N} - \frac{\Sigma X}{N}^2}$$

where X is the variable and N is the population.

For figuring the standard error of estimate the following equation was used for two variables:[3]

$$S_{xy} = S_x \sqrt{1 - r_{xy}^2}$$

where S_x is the standard deviation of the first variable and r is the simple correlation coefficient.

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[1] Ibid. p. 270.

[2] Guilford, op. cit., P. 91.

[3] Paul Blommers and E. R. Lindquist. Elementary Statistical Methods in Psychology and Education, p. 430.

The equation used for standard error of estimate for a three variable prediction was the following:[1]

$$S_{x.yz} = S_x \sqrt{1 - R^2_{x.yz}}$$

where S_x is the standard deviation of the first variable and $R_{x.yz}$ is the multiple correlation coefficient.

The following equation was used to compute the standard error of estimate between the predicted score and the achieved score:[2]

$$S_{yx} = \sqrt{\frac{\sum(Y - Y^1)^2}{N}}$$

where Y is the achieved score and Y^1 is the predicted score.

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[1]Guilford, *op. cit.*, p. 91.

[2]*Ibid.* p. 360.

OBSERVED TENDENCIES OF COLLECTED DATA FOR GEOMETRY

The following variables were investigated for predictive value for achievement in geometry.*

X_1 Grades of first year algebra, computed by the arithmetic mean of grades on ten chapter tests.

X_2 Grade point average (G.P.A.) of eighth and ninth grades (total).

X_3 Arithmetical Computation subtest of the S.R.A. High School Placement Test (H.S.P.T.).

X_4 Educational Ability (I.Q.) subtest of S.R.A. H.S.P.T.

Table I indicates the existence of certain relationships between the grades in algebra (X_1), grade point averages of eighth and ninth grades (X_2), S.R.A. Arithmetic Computation (X_3), and S.R.A. I.Q. (X_4), with the grade achievement in geometry (X_5).

X_1 (algebra I) and X_5 (geometry) were grades based on a twelve point system, where A=12, A-=11, B+=10,....., D-=2, and F=1. Components of variable X_2 were total grade averages for grades eight and nine based on a four point system where A=4, B=3, C=2, D=1, and F=0. Variable X_3 (H.S.P.T.) is represented by a percentile score and X_4 is the standard I.Q. score.

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*A table of all correlations considered for geometry are to be found in the appendix.

TABLE I

LISTING OF THE ACTUAL TEST RESULTS OF STUDENTS IN GEOMETRY

Student	Alg. I Grades X_1	G.P.A. of Fresh. X_2	H.S.P.T. X_3	I.Q. X_4	Geo. X_5
A1	11	3.8	99	134	11
A2	9	3.4	98	125	8
A3	8	3.0	75	117	7
A4	6	1.7	*	*	3
A5	8	2.2	*	*	3
A6	10	3.2	*	*	10
A7	6	2.2	*	*	3
A8	9	3.4	91	132	10
A9	8	2.5	*	*	11
A10	8	2.9	94	115	6
A11	8	2.8	*	*	6
A12	10	3.7	95	118	9
A13	8	3.1	98	116	8
A14	5	2.1	63	110	2
A15	5	2.4	43	100	5
A16	6	2.3	63	98	5
A17	10	2.7	*	*	12
A18	8	3.2	91	124	5
A19	5	2.1	78	98	2
A20	6	2.9	75	126	4
A21	8	3.2	85	121	8
A22	8	3.0	*	*	8
A23	9	3.6	94	105	10
A24	7	2.3	82	109	5
A25	8	3.0	99	128	9
A26	11	3.6	98	118	11
A27	8	3.1	91	123	8
A28	11	3.4	97	113	9
A29	8	2.8	60	107	7
A30	9	2.8	63	113	9
A31	5	2.7	94	123	5
A32	10	3.4	97	123	11
A33	10	2.7	91	109	8
A34	8	3.3	94	130	8
A35	10	3.6	99	122	10

*Test score not available.

A1, A26, and A35 are good examples of students whose scores are high in all instances. These students came from a favorable environmental background where maximum achievement was stressed. They were well motivated and interested in school work. A4, A15, A19, and A31 are examples of students who have relatively low grades in variables X_1 (algebra I), X_2 (G.P.A.), and X_5 (geometry). These students came from broken or alcoholic homes in which school achievement was not stressed. These students were left on their own and earned most of their money to go to school.

Although it is obvious there are relationships between the various test scores, it would seem highly desirable to have some statistical data to show these relationships. The most common method of showing a relationship between two variables is the coefficient of correlation. Table II shows the simple correlation between the variables for geometry.

TABLE II

SIMPLE CORRELATION FOR VARIABLES CONSIDERED FOR PREDICTING GRADES IN GEOMETRY

VARIABLES	ABBREVIATIONS	CORRELATIONS
X_1X_2	r_{12}	.7543
X_1X_5	r_{15}	.7871
X_2X_5	r_{25}	.7597
X_3X_5	r_{35}	.7058
X_4X_5	r_{45}	.1666

Variable X_3 (H.S.P.T.) was not used in this study for predictive purposes even though the coefficient of correlation was high enough to justify its inclusion ($r = .7058$). William Turnbull, executive vice president of the Educational Testing Service, said, "The test of arithmetic computation is a well constructed section that should provide a useful score on this important achievement area."[1] Also the manual of the H.S.P.T. suggests that one of the primary purposes of the test is to identify specific abilities of the student and also has value in predicting the students success in the various achievement areas.[2]

Variable X_4 (I.Q.) had a correlation coefficient of .1666. The writer believed this low correlation would be poor for predictive purposes. Possible explanation for this low correlation,

> is that students of low intelligence usually drop out of school at the first opportunity. Consequently high school students form a more homogeneous group than do elementary students, that is, there is less range in intellectual ability within the group. Thus it is harder to predict that one will do better than another.[3]

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[1]Oscar K. Buros (ed.) The Fifth Mental Measurements Yearbook, p. 58.

[2]Science Research Associates, Technical Report of the High School Placement Test, pp. 3-14.

[3]T. L. Engle, Psychology - Its Principles and Applications, World Book Company, p. 218.

Another reason the intelligence quotient had a low correlation was that other factors were involved such as interest, motivation, and aptitude. Also, another explanation was the small number of students with scores available for this study.

Grades of first year algebra computed by the arithmetic mean of ten chapter tests had a coefficient of correlation of .7871 with the geometry grades. The algebra chapter tests were taken from the booklet, Achievement Tests in First Algebra, by Virgil S. Mallory and William G. Koellner. The text book used was, First Algebra, by Virgil S. Mallory. The writer believed the test booklet was valid for prediction of geometry grades because of the high correlation. Roger Lennon supported this idea in the following statement,

> The validity of an aptitude, prognostic, or readiness test is the extent to which it is accurately indicates future learning success in the area for which it is used as a predictor (predictive validity). It is evidenced by correlations between test scores and measures of later success.[1]

Grade point average of the freshmen had a correlation of .7597 with geometry grades. The grade point averages of the freshmen were computed by averaging the final semester grade of subjects for which the student received graduation

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[1]Roger T. Lennon, "A Glossary of 100 Measurements Terms", Test Service Notebook, p. 6.

credit. The reason for this high correlation might be that the grade point averages are some indication of total achievement. This achievement could be relatively consistent in all classes, including mathematics.

Two of the criteria, X_1 (algebra I) and X_2 (G.P.A.), were considered for success in geometry. The first of these was the arithmetic mean of the grades of the ten chapter tests in algebra I ($r = .7871$). Further investigation showed variable X_1 (algebra I) had a low standard deviation of 1.293 and standard error of estimate 1.696 for X_1 (algebra I) with X_5 (geometry). The second measure of success was the freshmen grade point average ($r = .7597$). Investigation here showed that the standard deviation was .5196 and the standard error of estimate was 1.786 for X_1 (algebra I) with X_2 (G.P.A.). The reason for selection of these variables was the high correlation coefficient and low standard deviation and standard error of estimate. The writer felt that these measurements were of a high enough degree to justify their inclusion in this study.

The simple coefficients of correlation shown in Table II indicates the very high relationship between the selected variables (X_1X_5 and X_2X_5). With these correlations, the writer believed a student's success in geometry could be predicted by using these variables in computing the following

linear regression equations:

$$X_5' = 1.235X_1 - 2.734$$

$$X_5'' = 4.024X_2 - 4.451$$

where X_5' is the predicted score of geometry using variable X_1 (algebra I), and X_5'' is the predicted score of geometry using variable X_2 (G.P.A.).

The equations were computed by solving the equations for a and b.

Table III shows a comparison between the achieved scores and the predicted scores for geometry using the above equations. The standard error of estimate using X_5 (geometry) with X_5' (predicted geometry score using algebra I) was 1.4367. Using X_5 (geometry) with X_5'' (predicted geometry grade using G.P.A.) the standard error of estimate was 1.7672. A comparison of the predicted and the achieved variables showed a marked similarity.

As would be anticipated, correlations using two predicting variables were higher than the correlations using one predicting variable. The writer used X_1 (algebra I) and X_2 (G.P.A.) in computing the multiple correlation coefficient with X_5 (geometry). The multiple correlation coefficient was $R_{5.12} = .8267$ as compared with $r_{15} = .7871$ and $r_{25} = .7597$ for the simple correlation coefficients of these variables.

TABLE III

ACTUAL GRADE IN GEOMETRY AS COMPARED WITH PREDICTED GRADE BY THE LINEAR EQUATIONS

Student	Actual Score X_5	Predicted Score X_5'	Predicted Score X_5''
A1	11	10.9	10.8
A2	8	8.4	9.2
A3	7	7.2	7.6
A4	3	4.7	2.4
A5	3	7.2	4.4
A6	10	9.6	8.4
A7	3	4.7	4.4
A8	10	8.4	9.2
A9	11	7.2	5.6
A10	6	7.2	7.2
A11	6	7.2	6.8
A12	9	9.6	10.4
A13	8	7.2	8.0
A14	2	3.4	4.0
A15	5	3.4	5.2
A16	5	4.7	4.8
A17	11	9.6	6.4
A18	5	7.2	8.4
A19	2	3.4	4.0
A20	4	4.7	7.2
A21	8	7.2	8.4
A22	8	7.2	7.6
A23	10	8.4	10.0
A24	5	5.9	4.8
A25	9	7.2	7.6
A26	11	10.9	10.0
A27	8	7.2	8.0
A28	9	10.9	9.2
A29	7	7.2	6.8
A30	9	8.4	6.8
A31	5	3.4	6.4
A32	11	10.9	9.2
A33	8	10.9	6.4
A34	8	7.2	8.8
A35	10	10.9	10.0

With this high correlation, the writer felt that this was strong enough evidence to use the variables as predictors of geometry grades. The value of this predictor was also shown by the relatively low standard error of estimate of 1.548.

The following multiple regression equation was found by the previously described methods:

$$X_5''' = .7650X_1 + 2.0964X_2 - 5.0370$$

where X_5''' is the predicted score of geometry using algebra I (X_1) and G.P.A. (X_2).

Table IV shows the comparison of actual achieved scores and scores predicted by the above regression equation. There is a marked similarity between the scores.

The standard error of estimate for X_5''' with X_5 was 1.3973.

<u>Application of Obtained Data</u>: To illustrate the practical application of the results, the writer used regression equations to predict grades of students not included in the original study.

Table V shows the predicted scores and the actual score of each student. The writer felt that the results were very accurate.

<u>Summary</u>: From Table III and Table IV, the decimal places for the predicted scores should not be interpreted as being

TABLE IV

ACTUAL GRADE IN GEOMETRY AS COMPARED WITH PREDICTED GRADE USING THE MULTIPLE REGRESSION EQUATION

Student	Actual Score X_5	Predicted Score X_5'''
A1	11	11.4
A2	8	9.0
A3	7	7.4
A4	3	3.0
A5	3	5.7
A6	10	9.3
A7	3	4.2
A8	10	9.0
A9	11	6.3
A10	6	7.2
A11	6	7.0
A12	9	10.4
A13	8	7.6
A14	2	3.2
A15	5	3.8
A16	5	4.4
A17	11	8.3
A18	5	7.8
A19	2	3.2
A20	4	5.6
A21	8	7.8
A22	8	7.4
A23	10	9.4
A24	5	5.2
A25	9	7.4
A26	11	10.9
A27	8	7.6
A28	9	10.5
A29	7	7.0
A30	9	7.7
A31	5	4.4
A32	11	9.8
A33	8	8.3
A34	8	8.0
A35	10	10.2

TABLE V

PREDICTED GRADES OF GEOMETRY AS COMPARED WITH ACTUAL GRADES ON STUDENTS NOT USED IN THE ORIGINAL STUDY

Student	Actual Score X_5	Predicted Score X'_5	Predicted Score X''_5	Predicted Score X'''_5
D1	10	10.9	9.6	10.7
D2	7	7.2	8.0	7.5
D3	8	8.4	8.0	8.4
D4	9	8.4	7.6	8.1
D5	5	4.7	5.6	4.8
D6	7	7.2	8.0	7.5
D7	8	8.4	4.0	6.3
D8	7	7.2	6.8	7.0
D9	3	5.9	6.0	6.7
D10	10	9.6	9.6	10.0
D11	4	4.7	5.0	4.6
D12	6	5.9	6.8	6.2
D13	2	2.2	5.6	3.3
D14	10	9.6	8.8	9.5
D15	9	10.9	11.2	11.6
D16	9	8.4	8.4	9.3
D17	5	5.9	6.0	5.8
D18	6	2.2	6.4	4.5
D19	11	12.0	11.5	12.0
D20	6	5.9	7.2	6.4
D21	5	4.7	6.8	5.4
D22	10	10.9	11.5	11.8
D23	11	12.0	11.2	12.0
D24	7	7.2	7.8	7.4
D25	8	7.2	6.4	6.8
D26	7	8.4	7.8	8.1
D27	11	10.9	10.0	10.5
D28	6	7.2	7.8	7.4
D29	12	12.0	11.5	12.0
D30	11	12.0	11.5	12.0
D31	11	10.9	11.5	11.8
D32	8	8.4	6.8	7.5
D33	11	10.9	11.5	11.8
D34	10	10.9	11.5	11.8
D35	8	8.4	9.2	9.0

as accurate as they might appear. The writers purpose was not to make them appear accurate but to show the score as it would be derived from the equation. It would probably be better to round off to whole numbers.

The reader will note the apparent lack of relationship between the predicted score and the achieved score for students A5, A7, A9, A17, A18, and A25. The author felt that this relationship could be accounted for by several factors that were not considered in this statistical evaluation. Students A5, A7, and A18, who have a predicted score higher than their achieved score, had an apparent lack of motivation and interest. These students expressed dislike for the class and teacher because of a "personality conflict". Students A9, A17, and A25 had a predicted score lower than the achieved score. Even though their grade point averages were relatively low they showed a high ability for mathematics. These students were highly motivated and interested in the field of mathematics. If there would have been some measurements of these factors available, the writer felt that he could have made more reliable predictions.

It would appear that the best single predictor for success in geometry would be X_1 (grades in algebra I), because of the high correlation coefficient and low standard error of estimate. However, the difference in the predictive value of X_1 (algebra I) does not appear to be significantly higher than that for X_2 (G.P.A.).

OBSERVED TENDENCIES OF COLLECTED
DATA FOR ALGEBRA II

The following variables were investigated for predictive value for achievement in algebra II:*

- X_1 Grades of first year algebra computed by the arithmetic mean of grades on ten chapter tests.
- X_5 Grades of geometry computed by the arithmetic mean of grades on eleven chapter tests.
- X_6 Accumulative grade point average (G.P.A.) of sophomore students (grades 8, 9, and 10).
- X_7 Cooperative Geometry Test.
- X_8 California Short-Form Test of Mental Maturity, Advance Form S of 1957 (I.Q.).

Table VI indicates the existence of certain relationships between the grades in algebra I (X_1), grades of geometry (X_5), accumulative grade point average of sophomore students (X_6), Cooperative Geometry Test (X_7), and the California I.Q. (X_8), with the grade achievement in algebra II (X_9).

X_1 (algebra I), X_5 (Geometry), and X_9 (algebra II) are achieved grades based on the twelve point system. Components of variable X_6 are accumulative grade averages of sophomores based on the four point system. Variable X_7 is represented by the raw score for the Cooperative Geometry

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*A table of all correlations considered for algebra II are to be found in the appendix.

TABLE VI

LISTING OF THE ACTUAL TEST RESULTS
OF STUDENTS IN ALGEBRA II

Student	Grades Alg. I X_1	Grades Geo. X_5	G.P.A. Soph. X_6	Coop. Geo. Test X_7	Calif. I.Q. X_8	Grades Alg. II X_9
B1	11	11	4.0	33	118	11
B2	9	8	3.2	28	108	9
B3	8	7	2.7	28	98	4
B4	6	3	1.3	29	107	1
B5	10	10	3.2	29	103	8
B6	6	3	2.0	29	109	3
B7	9	10	3.8	31	119	7
B8	8	11	3.0	25	99	9
B9	8	6	3.0	26	103	4
B10	8	6	2.7	26	110	4
B11	10	9	3.3	26	102	9
B12	8	8	3.2	30	107	6
B13	5	5	2.3	27	109	1
B14	6	5	2.3	25	103	1
B15	10	11	3.0	28	113	10
B16	8	5	2.3	32	117	2
B17	6	4	2.5	26	118	2
B18	8	8	3.2	27	106	6
B19	8	8	3.0	31	109	5
B20	9	10	3.5	26	108	8
B21	7	5	2.0	27	110	3
B22	8	9	2.7	30	125	8
B23	11	11	3.8	27	105	8
B24	8	8	3.2	30	109	3
B25	11	9	3.5	29	114	10
B26	8	7	2.8	22	97	6
B27	9	9	3.2	33	107	10
B28	5	5	2.3	28	109	2
B29	10	11	3.7	29	111	10
B30	10	8	2.8	31	97	6
B31	8	8	3.5	27	119	6
B32	10	10	3.5	31	117	6

Test. X_8 is the standard I.Q.

B1, B15, B23, B25, B29, and B32 are good examples of students who are high in all instances. B4, B6, B13, B14, and B28 are examples of students who are relatively low in variables X_1 (algebra I), X_5 (geometry), X_6 (G.P.A.), and X_9 (algebra II).

To show the relationship between the various test scores a statistical analysis would be desirable. Table VII gives simple statistical data to show the relationship for algebra II.

TABLE VII

SIMPLE CORRELATION FOR VARIABLES CONSIDERED FOR PREDICTING GRADES IN ALGEBRA II

VARIABLES	ABBREVIATIONS	CORRELATIONS
X_1X_9	r_{19}	.9744
X_5X_9	r_{59}	.6430
X_6X_9	r_{69}	.7970
X_7X_9	r_{79}	.6360
X_8X_9	r_{89}	.1223

Variables X_5 (geometry) and X_7 (Cooperative Geometry Test) were not used in this study for predictive purposes even though the coefficient of correlation was relatively high (r_{59}=.6430 and r_{79}=.6360). The writer felt that in a

multiple regression equation of more than three variables these variables could be considered.

Variable X_8 (I.Q.) had a low correlation coefficient of .1223. Possible explanation for this low correlation coefficient would be the same as the S.R.A. I.Q. described in the section for geometry.

Two of the criteria, X_1 (algebra I) and X_6 (G.P.A.), were considered for success in algebra II. The text for algebra II was, Second Algebra, by Virgil S. Mallory and Kenneth C. Skeen. The chapter tests were taken from the booklet, Achievement Tests in Second Algebra, by the same authors.

Variable X_1 (algebra I) was selected because of the very high correlation coefficient which was .9744. Further investigation showed the standard deviation to be 1.192 and a standard error of estimate of .6827. The author felt that one possible explanation for the extremely high correlation was that both texts were written by the same author. It was observed that there was a great deal of review in the same book which would cause a continuation of the learning process. The presentation and reasoning of both texts were much the same.

Variable X_6 (G.P.A.) had a correlation coefficient of .7970. The standard deviation of this variable was .6000 and the standard error of estimate of X_9X_6 was 1.8362. The

writer believed the reason for the high correlation of this variable might be that students with high achievement that had selected mathematics were highly motivated and interested in this area. Because of the results of the statistical evaluation, variables X_1 (algebra I) and X_6 (G.P.A.) were selected as predictors for algebra II.

The use of the linear regression formula resulted in the following equations:

$$X_9' = 2.2894X_1 - 13.2987$$

$$X_9'' = 4.0358X_6 - 6.0431$$

where X_9' is the predicted score of algebra II using variables X_1 (algebra I) and X_9'' is the predicted score of algebra II using X_6 (G.P.A.).

Table VIII shows the similarity between the achieved scores and the predicted scores using the above equations. The standard error of estimate using X_9 (algebra II) with X_9' (predicted grade of algebra II using algebra I) was 1.8292. Using X_9 (algebra II) with X_9'' (predicted grade of algebra II using G.P.A.) the standard error of estimate was 1.7663. A comparison of the predicted and the achieved scores showed a marked similarity. As would be anticipated, the correlations using two predicting variables was higher than correlations using a single predictor. Variables X_1 (algebra I) and X_6 (G.P.A.) were used in computing the multiple correlation coefficient with X_9 (algebra II). The multiple

TABLE VIII

ACTUAL GRADE IN ALGEBRA II AS COMPARED WITH PREDICTED GRADE BY THE LINEAR EQUATIONS

Student	Actual Score X_9	Predicted Score X_9'	Predicted Score X_9''
B1	11	11.9	10.1
B2	9	7.3	6.9
B3	4	5.0	4.8
B4	1	1.0	1.0
B5	8	9.6	6.9
B6	3	1.0	2.0
B7	7	7.3	9.3
B8	9	5.0	6.1
B9	4	5.0	6.1
B10	4	5.0	4.8
B11	9	9.6	7.3
B12	6	5.0	6.9
B13	1	1.0	3.2
B14	1	1.0	3.2
B15	10	9.6	6.1
B16	2	5.0	3.2
B17	2	1.0	4.0
B18	6	5.0	6.9
B19	5	5.0	6.1
B20	8	7.3	8.1
B21	3	2.7	2.0
B22	8	5.0	4.8
B23	8	11.9	9.3
B24	3	5.0	6.9
B25	10	11.9	8.1
B26	6	5.0	5.3
B27	10	7.3	6.9
B28	2	1.0	3.2
B29	10	9.6	8.9
B30	6	9.6	5.3
B31	6	5.0	8.1
B32	6	9.6	8.1

correlation coefficient was $R_{9.16} = .9760$ as compared with $r_{91} = .9744$ and $r_{96} = .7970$.

With the extremely high multiple correlation coefficient and the low standard error of estimate, which was .6602, the writer felt this was strong enough evidence to justify the use of X_1 (algebra I) and X_6 (G.P.A.) to compute the following multiple regression equation:

$$X_9''' = 2.2171X_1 + .2027X_6 - 13.2920$$

where X_9''' is the predicted score using algebra I (X_1) and G.P.A. (X_6).

Table IX shows the comparison of the achieved scores and the scores predicted by the above equation. The standard error of estimate between X_9 (algebra II) and X_9''' was 1.7299.

<u>Application of Obtained Results</u>: To show the results of this chapter, the statistical data was applied to students not used in the original study.

Table X shows the results of the predicted grade with the actual grades using both linear regression equations and the multiple equation. The results were not favorable for accurate predictions of grades in algebra II. A possible explanation for the inconsistent prediction was the population was small.

In general, the predicted scores were lower than the

TABLE IX

ACTUAL GRADE IN ALGEBRA II AS COMPARED WITH PREDICTED GRADE USING THE MULTIPLE REGRESSION EQUATION

Student	Actual Score X_9	Predicted Score X_9'''
B1	11	11.9
B2	9	7.3
B3	4	4.9
B4	1	1.0
B5	8	9.5
B6	3	1.0
B7	7	7.4
B8	9	5.1
B9	4	5.1
B10	4	5.0
B11	9	9.5
B12	6	5.1
B13	1	1.0
B14	1	1.0
B15	10	9.5
B16	2	4.9
B17	2	1.0
B18	6	5.1
B19	5	5.0
B20	8	7.4
B21	3	2.6
B22	8	5.0
B23	8	11.8
B24	3	5.1
B25	10	11.8
B26	6	5.3
B27	10	7.3
B28	2	1.0
B29	10	9.6
B30	6	9.4
B31	6	5.2
B32	6	9.6

TABLE X

PREDICTED GRADES IN ALGEBRA II AS COMPARED WITH ACTUAL GRADES ON STUDENTS NOT USED IN THE ORIGINAL STUDY

Student	Actual Score X_9	Predicted Score X_9'	Predicted Score X_9''	Predicted Score X_9'''
E1	9	11.8	8.9	11.9
E2	9	9.6	6.5	9.5
E3	11	9.6	7.8	9.6
E4	2	1.0	3.2	1.0
E5	3	1.0	4.1	1.0
E6	5	1.0	4.9	1.0
E7	5	5.0	3.9	4.9
E8	9	9.6	7.7	9.5
E9	8	5.0	5.3	5.0
E10	11	11.8	10.1	12.0
E11	8	7.3	6.9	7.3
E12	10	9.6	9.3	9.7
E13	9	9.6	7.3	9.4
E14	9	9.6	8.9	9.6
E15	5	1.0	3.2	1.0
E16	7	5.0	5.3	5.0
E17	10	9.6	10.1	12.0
E18	7	7.3	5.3	7.2
E19	12	12.0	9.3	12.0
E20	12	11.0	9.3	12.0
E21	7	5.0	7.7	5.1
E22	9	9.6	5.3	9.4
E23	5	2.7	2.0	3.0
E24	9	11.8	9.7	12.0
E25	7	7.3	7.3	7.3
E26	4	2.7	2.0	2.6

actual score. One disadvantage of this would be that this might tend to discourage some of those students that would recieve passing grades.

Summary: Tables VII and VIII show apparent lack of relationship between the predicted score and the achieved score for students B2, B8, B16, B22, B23, B24, B27, B30, and B32. As was indicated in the summary of geometry the predictive scores are not to be taken as exact scores.

Students B16, B23, B24, B30, and B32 had a predicted score higher than the achieved score. Students B2, B8, B22, and B27 had a predicted score lower than the achieved score. The author believed this can be accounted for by noting students motivation, interest, abilities, and environmental background.

If a single predictor is to be used for the selection of students for algebra II it would appear, due to the high correlation coefficient, that variable X_1 (grades for algebraI) would produce the best results.

OBSERVED TENDENCIES OF COLLECTED DATA FOR SENIOR MATHEMATICS

The following variables were investigated for predictive value of achievement in senior mathematics:*

X_1 Grades of first year algebra I, computed by the arithmetic mean of grades on ten chapter tests.

X_5 Grades of geometry, computed by the arithmetic mean of grades on eleven chapter tests.

X_8 California Short-Form Test of Mental Maturity, Advance Form S of 1957 (I.Q.).

X_9 Grades of algebra II, computed by the arithmetic mean of grades on ten chapter tests.

X_{10} Accumulative junior grade point average (G.P.A.-eight, nine, ten, and eleven).

Table XI shows similarities between grades in algebra I (X_1), grades of geometry (X_5), California I.Q. (X_8), grades of algebra II (X_9), and accumulative grade point average of juniors (X_{10}) with grade achievement in senior mathematics (X_{11}).

X_1 (algebra I), X_5 (geometry), X_9 (algebra II), and X_{11} (senior mathematics) are achieved grades based on the twelve point system. Variable X_8 (I.Q.) is the standard I.Q. score. X_{10} is the junior grade point average based on the four point system.

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*A table of all correlations considered for senior mathematics are to be found in the appendix.

TABLE XI

LISTING OF THE TEST SCORES OF STUDENTS IN SENIOR MATHEMATICS

Student	Alg. I X_1	Geo. X_5	I.Q. X_8	Alg. II X_9	G.P.A. X_{10}	Sr. Math. X_{11}
C1	11	11	118	11	3.8	9
C2	10	10	103	8	3.2	8
C3	8	10	99	9	3.2	9
C4	10	11	113	10	3.0	8
C5	8	8	106	6	3.1	6
C6	8	8	109	5	2.9	3
C7	8	8	125	8	2.7	7
C8	11	11	105	8	3.5	7
C9	11	10	114	10	3.6	8
C10	11	11	111	10	3.4	9
C11	10	8	97	6	2.8	6
C12	8	8	119	6	3.5	7
C13	10	10	117	6	3.5	6
C14	8	3	116	7	2.3	2
C15	11	8	123	10	2.9	8
C16	8	8	95	6	2.8	5
C17	6	4	108	4	2.5	2
C18	12	12	106	11	3.8	11
C19	10	10	112	10	3.3	10
C20	12	12	124	12	4.0	12
C21	11	10	126	10	3.6	11
C22	12	12	*	12	3.8	11
C23	11	12	117	11	4.0	11
C24	11	10	111	10	3.8	9

*Test score not available.

Table XII shows the simple correlation coefficients between the variables for senior mathematics.

TABLE XII

SIMPLE CORRELATIONS OF THE CONSIDERED VARIABLES WITH SENIOR MATHEMATICS

VARIABLES	ABBREVIATION	CORRELATIONS
X_1X_{11}	$r_{1.11}$	.7973
X_5X_{11}	$r_{5.11}$	.8652
X_8X_{11}	$r_{8.11}$	.1166
X_9X_{11}	$r_{9.11}$	.8869
$X_{10}X_{11}$	$r_{10.11}$	.8422

Variables X_1 (algebra I) and X_{10} (G.P.A.) were not used for predictive purposes in this study although their correlation coefficients ($r_{1.11}$=.7973 and $r_{10.11}$=.8422) would suggest they could be used. The only reason the writer did not use these variables was because variables X_5 (geometry) and X_9 (algebra II) had a higher correlation coefficients. It would have been desirable to use these variables in a multiple regression problem of three or more variables.

The California I.Q. (X_8) was not used because of its low correlation. The possible reason for the low correlation were explained in the previous sections on geometry and algebra II.

The text for senior mathematics used at Larned Senior High School was Advanced High School Mathematics by Glen D. Vannatta, Walter H. Carnahan, and Harold P. Fawcett. The chapter tests were taken from the booklet, Test Booklet for Advanced High School Mathematics by G. Lawerence Hopkins.

The two criteria considered for success in predicting grades for senior mathematics were grades of geometry (X_5) and grades of algebra II (X_9). These variables were selected because of their correlations ($r_{5.11}$=.8652 and $r_{9.11}$=.8869), standard deviations (s_5=2.2690 and s_9=2.2913), and standard error of estimate ($s_{11.5}$=1.3689 and $s_{11.9}$=1.2611). With the advancement through the mathematics program, the writer felt that with the deletion of the poorer students that any mathematics class would be a good predictor. Most students that were not interested, motivated, talented or "directed" would be eliminated by the time they reached the senior mathematics class.

With the use of linear regression formula the following equations were calculated:

$$X'_{11} = 1.0404X_5 - 2.0454$$

$$X''_{11} = 1.0576X_9 - 1.3694$$

where X'_{11} is the predicted senior mathematics score using variable X_5 (geometry), and X''_{11} (senior mathematics) is the predicted score using variable X_9 (algebra II).

Table XIII shows the computed results of the preceeding equations. The standard error of estimate between the predicted grades of senior mathematics (X'_{11}) and the actual grades (X_{11}) was 1.2499. For X''_{11} with X_{11}, the standard error of estimate was 1.3758.

Using two predicting variables the correlation coefficient was higher than those of the single predictor. The multiple correlation coefficient was $R_{11.59}=.9386$. Because of the high multiple correlation coefficient and the low standard error of estimate ($s_{11.59}=.3253$) the multiple regression formula was used to derive the following equation:

$$X'''_{11} = .5525X_5 + .6504X_9 - 3.0540$$

where X'''_{11} is the predicted score of senior mathematics using geometry and algebra II.

Table XIV shows the results of the above equation with the actual score obtained in senior mathematics. The standard error of estimate between the actual score and the predicted score was .9527.

<u>Application of Obtained Data</u>: Table XV contains results derived by using the linear and multiple regression equations on students not used in the original study. The table shows the actual score with the predicted scores for students in senior mathematics. The scores showed a high relationship,

TABLE XIII

ACTUAL GRADES IN SENIOR MATHEMATICS AS COMPARED WITH PREDICTED GRADES FROM THE LINEAR REGRESSION EQUATIONS

Student	Actual Score X_{11}	Predicted Score X'_{11}	Predicted Score X''_{11}
C1	9	10.3	9.4
C2	8	7.1	8.4
C3	9	8.2	8.4
C4	8	9.2	9.4
C5	6	5.0	6.3
C6	3	3.9	6.3
C7	7	7.1	6.3
C8	7	7.1	9.4
C9	8	9.2	8.4
C10	9	9.2	9.4
C11	6	5.0	6.3
C12	7	5.0	6.3
C13	6	5.0	8.4
C14	2	6.0	1.0
C15	8	9.2	6.3
C16	5	5.0	6.3
C17	2	2.9	2.1
C18	11	10.3	10.4
C19	10	9.2	8.4
C20	12	11.3	10.4
C21	11	9.2	8.4
C22	11	11.3	10.4
C23	11	10.3	10.4
C24	9	9.2	8.4

TABLE XIV

ACTUAL GRADES IN SENIOR MATHEMATICS AS COMPARED WITH PREDICTED GRADES FROM THE MULTIPLE REGRESSION EQUATION

Student	Actual Score X_{11}	Predicted Score X'''_{11}
C1	9	10.2
C2	8	7.8
C3	9	8.4
C4	8	9.6
C5	6	5.5
C6	3	4.9
C7	7	6.6
C8	7	8.5
C9	8	9.0
C10	9	9.6
C11	6	5.5
C12	7	5.5
C13	6	6.8
C14	2	2.8
C15	8	7.7
C16	5	5.5
C17	2	1.8
C18	11	10.8
C19	10	9.0
C20	12	11.4
C21	11	9.0
C22	11	11.4
C23	11	10.8
C24	9	9.0

TABLE XV

PREDICTED GRADES FOR SENIOR MATHEMATICS AS COMPARED WITH ACTUAL GRADES FOR STUDENTS NOT USED IN THE ORIGINAL STUDY

Student	Actual Score X_{11}	Predicted Score X'_{11}	Predicted Score X''_{11}	Predicted Score X'''_{11}
F1	8	7.3	8.1	7.8
F2	8	7.3	8.1	7.8
F3	9	8.4	10.2	9.6
F4	1	3.1	1.0	1.2
F5	8	9.4	8.1	8.7
F6	1	3.2	1.0	1.6
F7	5	6.3	3.9	4.6
F8	5	5.2	5.0	4.7
F9	9	9.4	8.1	8.7
F10	7	6.3	7.1	6.6
F11	11	10.4	10.2	10.7
F12	8	9.4	7.1	8.5
F13	10	9.4	9.2	9.5
F14	9	9.4	8.1	9.0
F15	8	9.4	8.1	9.0
F16	5	5.2	3.9	4.0
F17	6	6.3	6.0	5.9
F18	9	9.4	9.2	9.6
F19	7	6.3	6.0	6.0
F20	12	10.5	11.3	11.4
F21	11	10.5	10.2	10.8
F22	5	5.2	6.0	5.4
F23	7	5.2	8.1	6.5
F24	4	4.2	3.9	3.5
F25	5	5.2	5.0	4.7
F26	8	9.4	8.1	9.0
F27	6	7.3	6.0	6.5
F28	1	1.0	2.9	1.0

therefore, the writer felt that these equations could be used as predictors for future cases.

Summary: From Tables XIII and XIV the reader will note a slight amount of disagreement between the predicted score and the achieved score for students C4, C6, C12, and C21. Students C4 and C6 have a predicted score higher than their achieved score, while students C12 and C21 have a lower predicted score than achieved score.

A relatively accurate prediction of grades for senior mathematics should be obtained by using any one of the regression equations. The low standard error of estimate would indicate that the multiple regression equation would give the most valid prediction. Having a knowledge of the students background would lead to more accurate predictions.

SUMMARY AND CONCLUSIONS

The purpose of this study was to find a method for predicting grades of geometry, algebra II, and senior mathematics to reduce the amount of misplacement in these classes. The data for this study was obtained from permanent records and the mathematics department of the Larned Senior High School. The method used for predicting was done in the form of linear and multiple regression equations. The variables used in these equations were selected by determining the correlation coefficients with the grades in the various mathematics classes of Larned Senior High School. The standard deviations and standard error of estimates were computed for all considered variables. Tables were used to show the actual scores and the derived results. Further tables were made for showing the accuracy of the final results on students not included in the original study.

It was found in geometry (X_5) that the best single predictor was algebra I (X_1). The equation derived by using this variable was:

$$X_5' = 1.235X_1 - 2.734$$

Greater accuracy was found by using the multiple regression equation derived by using algebra I (X_1) and

freshman grade point average (X_2). The equation was:

$$X_5''' = .7650X_1 + 2.0964X_2 - 5.0370$$

The multiple correlation coefficient was higher and the standard error of estimate was somewhat smaller.

The best single predictor for algebra II (X_9) was grades of algebra I (X_1). The correlation coefficient for these two variables was extremely high (.9744). The linear equation was:

$$X_9' = 2.2894X_1 - 13.2987$$

The multiple regression equation was computed using algebra I (X_1) and sophomore grade point average (X_6). These variables were selected because they had the highest correlation coefficients. The results of this equation:

$$X_9''' = 2.2171X_1 + .2027X_6 - 13.2920$$

were not significantly better than those of the single predictor. As would be expected the multiple correlation coefficient was higher than that of the single predictor.

In the study of senior mathematics, the regression equations using grades of geometry (X_5) and algebra II (X_9) with grades of senior mathematics (X_{11}) were found to be

good predictors. These equations were as follows:

$$X'_{11} = 1.0404X_5 - 2.0454$$
$$X''_{11} = 1.0576X_9 - 1.3694$$
$$X'''_{11} = .6504X_9 + .5525X_5 - 3.0540$$

The greatest accuracy was obtained from the multiple regression equation. This was shown by the higher multiple correlation and significantly lower standard error of estimate.

Of all the variables considered in this study, algebra I had the highest correlation with geometry and algebra II. A possible explanation for these high correlations was that the books used for these classes were a series by the same author. The books used the same approach to present the subject matter which would lead to a continuation of the learning process.

The next highest correlation for geometry and algebra II were freshman and sophomores grade point averages, respectively. The grade point average should give a measure of total achievement which should be relatively the same for all classes including mathematics. The grade point average is influenced by the properties of motivation, interest, study habits and environmental background.

The best correlations for senior mathematics were grades of geometry and algebra II. The senior mathematics book used was selected by the school because it followed the same approach and line of reasoning as the previous series.

In considering the results of this study, it must be realized that the population was small and applied only to Larned Senior High School. The types of variables that were available for this study were limited. Some of the results missing were aptitude tests, interest tests, and achievement tests. Some of the records that were available could not be used because of missing results. If there would have been a larger population and more variables available, it is probable that the results would have been more significant.

Recommendations: The writer felt that the results of this study could be used to prevent misplacement of students. A counselor could use the predicted grades to show the probable achievement of the student. A student with a low predicted score could be shown the possible difficulties of entering a certain mathematics class.

Also, the study showed that there were weaknesses in the testing program of the guidance department of Larned Senior High School. There seemed to be a need for mathematical aptitude and achievement tests. As a result of this study, the guidance department has obtained the California

Algebra Aptitude Test to be administered at the eighth grade level.

There was some need to provide a program to fit the needs of these misplaced students. The type of program should be an accelerated mathematics program to meet the needs of college bound students and mathematics program to meet the needs of everyday living. This was brought to the attention of the administration. For the school year of 1964-1965, a general mathematics class was incorporated into the program of studies. Also, there was some consideration given by the principal to dividing algebra II and geometry into two classes, one being terminating and the other for college bound students.

A continued study should be conducted each year for the best results in predicting grades of students in mathematics classes. This would tend to make the regression equations more accurate due to the increase number of scores.

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APPENDIX

Tables XVI, XVII, and XVIII were set up to provide the necessary information for use in the computer. It could easily provide a basis for further investigation.

The following is a list of variables that were investigated through correlations with geometry:

1. Geometry Grades (Geo.)
2. Algebra I Grades (Alg. I)
3. Grade Point Average of Freshman (GPA-F)
4. Differential Aptitude Tests - Numerical Ability (DAT-NA)
5. Space Relations of the S.R.A. I.Q. test. (IQ-SR)
6. Science Research Associates Intelligence Test (I.Q.)

Table XVI shows the correlations between all the investigated variables.

TABLE XVI

THE RELATIONSHIP OF THE VARIABLES INVESTIGATED FOR PREDICTIVE PURPOSES OF GEOMETRY

	Geo.	Alg. I	GPA-F	DAT-NA	I.Q.-SR	I.Q.
Geo.	1.00	.79	.76	.52	.22	.16
Alg. I		1.00	.75	.43	.01	.03
GPA-F			1.00	.50	.38	.31
DAT-NA				1.00	.63	.26
I.Q.-SR					1.00	.17
I.Q.						1.00

The following is a list of variables that were investigated through correlations with algebra II:

1. Algebra II Grades (Alg.II)
2. Algebra I Grades (Alg.I)
3. Grade Point Average of Sophomores (GPA-S)
4. Geometry Grades (Geo.)
5. Cooperative Geometry Test (Coop. Geo.)
6. Grade Point Average of Freshman (GPA-F)
7. California Test of Mental Maturity (I.Q.)
8. Differential Aptitude Test - Numerical Ability (DAT-NA)

Table XVII shows the correlations between all investigated variables.

TABLE XVII

THE RELATIONSHIP OF VARIABLES INVESTIGATED FOR PREDICTIVE PURPOSES OF ALGEBRA II

	Alg. II	Alg. I	GPA-S	Geo.	Coop. Geo.	GPA-F	I.Q.	DAT-NA
Alg. II	1.00	.97	.80	.64	.64	.62	.12	.42
Alg. I		1.00	.77	.79	.21	.72	.04	.30
GPA-S			1.00	.86	.20	.87	.17	.44
Geo.				1.00	.21	.69	.07	.38
Coop. Geo.					1.00	.22	.48	.03
GPA-F						1.00	.27	.38
I.Q.							1.00	.23
DAT-NA								1.00

The following is a list of variables that were investigated through correlations with senior mathematics:

1. Senior Mathematics Grades (S.M.)
2. Algebra II Grades (Alg.II)
3. Cooperative Algebra Test (Coop. Alg.)
4. Differential Aptitude Test - Numerical Ability (DAT-NA)
5. Grade Point Average of Juniors (GPA-J)
6. Algebra I Grades (Alg. I)
7. Geometry Grades (Geo.)
8. National Merit Scholarship Qualifying Test (N.M.)

Table XVIII shows the correlations between all investigated variables.

TABLE XVIII

THE RELATIONSHIP OF VARIABLES INVESTIGATED FOR PREDICTIVE PURPOSES OF SENIOR MATHEMATICS

	S.M.	Alg. II	Coop. Alg.	DAT-NA	GPA-J	Alg. I	Geo.	N.M.
S.M.	1.00	.89	.66	.67	.84	.80	.87	.41
Alg. II		1.00	.56	.57	.73	.83	.85	.39
Coop. Alg.			1.00	.48	.57	.49	.55	.39
DAT-NA				1.00	.42	.60	.57	.18
GPA-J					1.00	.77	.86	.18
Alg. I						1.00	.80	.40
Geo.							1.00	.47
N.M.								1.00

PREDICTING SUCCESS IN GEOMETRY, ALGEBRA II, AND SENIOR MATHEMATICS AT LARNED HIGH SCHOOL

by

JAMES C. SHORTT, JR.

B. S., Kansas State University, 1961

AN ABSTRACT OF A MASTER'S REPORT

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Across our nation there has been a growing emphasis placed on mathematics in our schools. This has presented problems to our schools as well as to many students. These problems have added to the responsibilities of administrators and counselors to help students overcome and adjust to their difficulties in mathematics courses.

The purpose of this study was to predict success of geometry, algebra II, and senior mathematics for students at Larned High School. This would help limit the misplacement of students, thus preventing a number of problems from developing.

The data for this study was obtained from the permanent records and the mathematics department of Larned High School for the school year 1962-1963. The method used for predicting was done in the form of linear and multiple regression equations. The variables used in these equations were selected by determining the correlation coefficients with the grades of the various mathematics classes. Tables were used to show the actual scores and the scores derived from the regression equations. Other tables were used to show the accuracy of the final results on students not used in the original study.

It was found in geometry (X_5) that the best single predictor was algebra I (X_1). The correlation coefficient was .78. The equation derived by using this variable was

as follows:

$$X_5' = 1.235X_1 - 2.734$$

Greater accuracy was found by using the multiple regression equation derived by using algebra I (X_1) and freshman grade point average (X_2). The equation was:

$$X_5''' = .7650X_1 + 2.0964X_2 - 5.0370$$

The multiple correlation coefficient was higher ($R=.83$) than the simple correlations.

The best single predictor for algebra II (X_9) was grades of algebra I (X_1). The correlation coefficient for these two variables was extremely high ($r=.974$). The linear regression equation was:

$$X_9' = 2.2894X_1 - 13.2987$$

The multiple regression equation was computed by using algebra I grades (X_1) and sophomore grade point averages (X_6). These variables were selected because they had the highest correlation coefficients. The equation was:

$$X_9''' = 2.2171X_1 + .2027X_6 - 13.2920$$

The multiple correlation ($R=.976$) was not significantly higher than that of the single predictor.

In the study of senior mathematics, the regression equations using grades of geometry (X_5) and algebra II (X_9) were found to be good predictors. These equations were:

$$X'_{11} = 1.0404X_5 - 2.0454$$

$$X''_{11} = 1.0576X_9 - 1.3694$$

The greatest accuracy was found by using the following multiple regression equation:

$$X'''_{11} = .6504X_9 + .5525X_5 - 3.0540$$

It would seem from this study that future success in mathematics classes at Larned High School can best be predicted from grades of past mathematics classes and past grade point averages.